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P. Salmon

THE SCIENTIFIC WORK OF PAOLO VALABREGA

Dedicated to Paolo Valabrega on the occasion of his 60 th birthday

Paolo Valabrega got his degree as "dottore in Scienze Matematiche" in the year 1968, discussing a thesis on some problems related to general topology under the direction of D. Demaria. He published soon a first article on a topological subject and, almost immediately later, a second one of historical interest on Hilbert's "Grundlagen der Geometrie" in cooperation with E. Valabrega.

Starting with 1969, the main scientific interest of Paolo V. moved to two subjects, streectly connected with each other: Commutative Algebra and Algebraic Geometry. More precisely, in the period 1969–1980, Paolo V. studied essentially problems related with Commutative Algebra, while, starting 1980, the main interest became Algebraic Geometry, mainly algebraic curves.

The first contributions of Paolo V. to problems in Commutative Algebra are related with the scientific activity promoted, first of all in Pisa and then in Genova, as a consequence of my personal experience in Paris, working there, in the years 1962 and 1963, under the scientific direction of P. Samuel. In that period P. Samuel was particularly interested in factorial rings, and he could exibhite, among several results, the first well known examples of non factorial power series rings over a basic factorial ring. My work in Paris did concern essentially the factoriality of restricted power series rings over a regular and factorial ring A with respect to a suitable $\mathfrak{m}$-adic topology fixed in A. In the following years, Paolo V. too was inerested in restricted power series.

When, in the summer of 1969, Paolo Valabrega came to Genova with the proposal of a collaboration with our group working in Commutative Algebra, among the people inside that group (living in Genova and outside) one may mention: P. Corsini, M. Fiorentini, S. Greco, S. Guazzone, T. Millevoi, C. Pedrini, L. Robbiano, P. Salmon, G. Valla, G. Vecchio. The principal objects in their research in Commutative Algebra were: factorial rings, algebraic properties of $\mathfrak{m}$-adic completions, henselian rings and their first generalizations by Lafon and Greco, projective modules, Picard groups, symmetric algebras and Rees algebras related to ideals. D. Gallarati and V. Villani, both professors of Geometry in Genova and interested respectively in Algebraic Geometry and Complex Geometry, very often offered their cooperation with the Commuative Algebra group: some of their students did work on problems of common interest.

The starting point of the first research developed by Paolo V. in the field of Commutative Algebra was the classical Hensel's Lemma, stating that if $(A, \mathfrak{m})$ is a complete local ring and f is a unitary polynomial in the polynomial ring $A[X]$ with image f^* in $(A/\mathfrak{m})[X]$, a decomposition of f^* in a product of relatively prime polynomials in $(A/\mathfrak{m})[X]$ can be lifted to a decomposition of f in $A[X]$. In Chapter 4 of Bourbaki's Algèbre Commutative, appeared in 1961, there was a generalization of the above statement involving, beside a unitary polynomial, also restricted power series rings with respect to linear topologies where the basic commmutative ring A is not necessarily local and the role of the maximal ideal in the classical statement it was assumed by a closed ideal $\mathfrak{m}$ of A.

This generalization suggested to Paolo V. some investigations related with the new Hensel's Lemma and he was able to write three articles (appeared around 1972) concerning properties of henselian topological rings, topological henselizations and further extentions with applications to valuation rings.

The articles written by Paolo V. in the following years 1972–1980 are primarily concerned with excellent rings, starting with statements of the excellent property in some topological rings: convergent series and restricted power series rings, completions of excellent rings (one article was written in collaboration with S. Greco). A result, particularly appreciated by H. Matsumura, states the excellent property for the ring of formal power series over a polynomial ring.

In the same period Paolo V. published also some articles on openness of loci in a particular scheme, lifting properties and special morphisms (one article was in collaboration with C. Massaza).

In the years 1978–1980 Paolo V. wrote two articles, working together with G. Valla, on graded rings (form rings and regular sequences, standard bases). Both articles were studied and highly appreciated by several algebraic experts in the field, also in recent years.

Starting with 1980, Paolo V. focused his interest on subjects more specifically related with Algebraic Geometry. In that direction he started a collaboration with S. Greco on the theory of adjoint curves and, later, on the singularities of algebraic varieties containing a fixed algebraic curve.

Then, Paolo V.'s interest in Algebraic Geometry had a further and permanent increasing, with a particular interest for subcanonical curves. He wrote on that subject several articles, sometimes in collaboration with L. Chiantini, A. Geramita and M. Roggero.

Going on, the collaboration with M. Roggero, which began before 1990, had a resolute prominence in the following years and until recent times; it concerned subcanonical curves and surfaces, vector bundles, reflexive sheaves and complete intersections.

Paolo SALMON, Dipartimento di Matematica, Università di Bologna, Piazza di Porta S. Donato 5 - 40126 Bologna - ITALIA
e-mail: `salmon@neu.edu`